1. An and statement is true if, and only if, both components are __.

2. An or statement is false if, and only if, both components are __.

3. Two statement forms are logically equivalent if, and only if, they always have __.

4. De Morgan’s laws say (1) that the negation of an and statement is logically equivalent to the ___ statement in which each component is __, and (2) that the negation of an or statement is logically equivalent to the ___ statement in which each component is __.

In each of 5–8 represent the common form of each argument using letters to stand for component sentences, and fill in the blanks so that the argument in part (b) has the same logical form as the argument in part (a).

5. a. If all integers are rational, then the number 1 is rational.
   All integers are rational.
   Therefore, the number 1 is rational.
   b. If all algebraic expressions can be written in prefix notation, then ____________________________.
   ____________________________.
   Therefore, \((a + 2b)(a^2 - b)\) can be written in prefix notation.

6. a. If all computer programs contain errors, then this program contains an error.
   This program does not contain an error.
   Therefore, it is not the case that all computer programs contain errors.
   b. If __, then __.
   2 is not odd.
   Therefore, it is not the case that all prime numbers are odd.

7. a. This number is even or this number is odd.
   This number is not even.
   Therefore, this number is odd.
   b. __ or logic is confusing.
   My mind is not shot.
   Therefore, __.

8. a. If \(n\) is divisible by 6, then \(n\) is divisible by 3.
   If \(n\) is divisible by 3, then the sum of the digits of \(n\) is divisible by 3.
   Therefore, if \(n\) is divisible by 6, then the sum of the digits of \(n\) is divisible by 3.
   (Assume that \(n\) is a particular, fixed integer.)
   b. If this function is ___ then this function is differentiable.
   If this function is ___ then this function is continuous.
   Therefore, if this function is a polynomial, then this function __.
9. Let $p$ be the statement “DATAENDFLAG is off,” $q$ the statement “ERROR equals 0,” and $r$ the statement “SUM is less than 1,000.” Express the following sentences in symbolic notation.
   a. DATAENDFLAG is off, ERROR equals 0, and SUM is less than 1,000.
   b. DATAENDFLAG is off but ERROR is not equal to 0.
   c. DATAENDFLAG is off; however, ERROR is not 0 or SUM is greater than or equal to 1,000.
   d. DATAENDFLAG is on and ERROR equals 0 but SUM is greater than or equal to 1,000.
   e. Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

10. Write truth tables for the statement forms for $p \land (q \land r)$.

11. Are $p \land (q \land r)$ and $(p \land q) \land r$ logically equivalent?

12. A logical equivalence is derived from Theorem 2.1.1. Supply a reason for each step.

\[
(p \land \sim q) \lor (p \land q) \equiv p \land (\sim q \lor q) \quad \text{by (a)}
\]
\[
\equiv p \land (q \lor \sim q) \quad \text{by (b)}
\]
\[
\equiv p \land t \quad \text{by (c)}
\]
\[
\equiv p \quad \text{by (d)}
\]

Therefore, $(p \land \sim q) \lor (p \land q) \equiv p$.

13. Use Theorem 2.1.1 to verify the logical equivalences.
   a. $(p \land \sim q) \lor p \equiv p$
   b. $\sim ((\sim p \land q) \lor (\sim p \land \sim q)) \lor (p \land q) \equiv p$