Use modus ponens or modus tollens to fill in the blanks in the arguments of 1–2 so as to produce valid inferences.

1. If \( \sqrt{2} \) is rational, then \( \sqrt{2} = \frac{a}{b} \) for some integers \( a \) and \( b \).
   It is not true that \( \sqrt{2} = \frac{a}{b} \) for some integers \( a \) and \( b \).
   \[ \therefore \]

2. If \( 1 - 0.99999 \ldots \) is less than every positive real number, then it equals zero.
   \[ \therefore \] The number \( 1 - 0.99999 \ldots \) equals zero.

Use truth tables to determine whether the argument forms in 3–4 are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid or invalid.

3. \( p \rightarrow q \)
   \( q \rightarrow p \)
   \[ \therefore p \lor q \]

4. \( p \)
   \( p \rightarrow q \)
   \( \sim q \lor r \)
   \[ \therefore r \]

Some of the arguments in 5–10 are valid, whereas others exhibit the converse or the inverse error. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

5. If Jules solved this problem correctly, then Jules obtained the answer 2.
   Jules obtained the answer 2.
   \[ \therefore \] Jules solved this problem correctly.

6. This real number is rational or it is irrational.
   This real number is not rational.
   \[ \therefore \] This real number is irrational.

7. If I go to the movies, I won’t finish my homework. If I don’t finish my homework, I won’t do well on the exam tomorrow.
   \[ \therefore \] If I go to the movies, I won’t do well on the exam tomorrow.

8. If this number is larger than 2, then its square is larger than 4.
   This number is not larger than 2.
   \[ \therefore \] The square of this number is not larger than 4.

9. In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a–e below) and challenged the reader to use them to figure out the location of the treasure.
   a. If this house is next to a lake, then the treasure is not in the kitchen.
   b. If the tree in the front yard is an elm, then the treasure is in the kitchen.
   c. This house is next to a lake.
   d. The tree in the front yard is an elm or the treasure is buried under the flagpole.
   e. If the tree in the back yard is an oak, then the treasure is in the garage.
   Where is the treasure hidden?

10. Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:
    a. Socko: Lefty killed Sharky.
    b. Fats: Muscles didn’t kill Sharky.
    c. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
    d. Muscles: Lefty didn’t kill Sharky.
    Who did kill Sharky?
In 11 and 12 a set of premises and a conclusion are given. Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in Example 2.3.8. Assume all variables are statement variables.

11. a. ¬p ∨ q → r  
b. s ∨ ¬q  
c. ¬t  
d. p → t  
e. ¬p ∧ r → ¬s  
f. ∴ ¬q

12. a. ¬p → r ∧ ¬s  
b. t → s  
c. u → ¬p  
d. ¬w  
e. u ∨ w  
f. ∴ ¬t