Write 1 using summation or product notation.
1. \(1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + 7^2 - 8^2\)

Transform 2 by making the change of variable \(i = k + 1\).
2. \(\sum_{k=0}^{5} k(k - 1)\)

Transform 3 by making the change of variable \(j = i - 1\).
3. \(\sum_{i=1}^{n+1} \frac{(i-1)^2}{i \cdot n}\)

4. Prove that for all nonnegative integers \(n\) and \(r\) with \(r + 1 \leq n\),
\[
\binom{n + 1}{r + 1} = \frac{n + 1}{r + 1} \cdot \binom{n}{r}
\]

Prove statement 5 by mathematical induction.
5. \(1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}\), for all integers \(n \geq 1\).

Use the formula for the sum of the first \(n\) integers and/or the formula for the sum of a geometric sequence to evaluate the sums in 6-7 or to write them in closed form.
6. \(4 + 8 + 12 + 16 + \cdots + 200\)

7. \(1 - 2 + 2^2 + \cdots + (-1)^n 2^n\), where \(n\) is a positive integer.

8. Tower of Hanoi with Adjacency Requirement: Suppose that in addition to the requirement that they never move a larger disk on top of a smaller one, the priests who move the disks of the Tower of Hanoi are also allowed only to move disks one by one from one pole to an adjacent pole. Assume poles A and C are at the two ends of the row and pole B is in the middle. Let \(a_n\) = the minimum number of moves needed to transfer a tower of \(n\) disks from pole A to pole B.
   a. Find \(a_1, a_2, a_3\),
   b. Find a recurrence relation for \(a_1, a_2, a_3, \ldots\)

For 9-10, use iteration to guess an explicit formula for the sequence.
9. \(a_k = k \cdot a_{k-1}\), for all integers \(k \geq 1\), \(a_0 = 1\)

10. \(s_k = s_{k-1} + k\), for all integers \(k \geq 1\), \(s_0 = 3\)

11. Tower of Hanoi with Adjacency Requirement: Suppose that in addition to the requirement that they never move a larger disk on top of a smaller one, the priests who move the disks of the Tower of Hanoi are also
allowed only to move disks one by one from one pole to an adjacent pole. Assume poles A and C are at the two ends of the row and pole B is in the middle. Let
\[ a_n = \text{the minimum number of moves needed to transfer a tower of } n \text{ disks from pole A to pole C}. \]
a. Find \( a_1, a_2, a_3 \),
b. Find a recurrence relation for \( a_1, a_2, a_3, \ldots \)

12. Find the explicit formula for \( a_k \) in Problem 11.