1. Which of the following is a negation for “All discrete mathematics students are athletic”?
   More than one answer may be correct.
   a. There is a discrete mathematics student who is nonathletic.
   b. All discrete mathematics students are nonathletic.
   c. There is an athletic person who is a discrete mathematics student.
   d. No discrete mathematics students are athletic.
   e. Some discrete mathematics students are nonathletic.
   f. No athletic people are discrete mathematics students.

2. Let D = {−48, −14, −8, 0, 1, 3, 16, 23, 26, 32, 36}. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.
   a. ∀x ∈ D, if x is less than 0 then x is even.
   b. ∀x ∈ D, if the ones digit of x is 2, then the tens digit is 3 or 4.
   c. ∀x ∈ D, if the ones digit of x is 6, then the tens digit is 1 or 2.

   In 3 determine whether the proposed negation is correct. If it is not, write a correct negation.

3. Statement: The sum of any two irrational numbers is irrational.

   Proposed negation: The product of any irrational number and any rational number is rational.

4. For all squares x there is a circle y such that x and y have different colors and y is above x.
Each statement in 5–6 refers to the above Tarski world. For each, (a) determine whether the statement is true or false and justify your answer, (b) write a negation for the statement.

5. \( \forall \) circles \( x \) and \( \forall \) triangles \( y \), \( x \) is above \( y \).

6. \( \exists \) a triangle \( x \) and \( \exists \) a square \( y \) such that \( x \) is above \( y \) and \( x \) and \( y \) have the same color.

In 7-8, refer to the Tarski world given above. The domains of all variables consist of all the objects in the Tarski world. For each statement, (a) indicate whether the statement is true or false and justify your answer, (b) write the given statement using the formal notation, and (c) write the negation of the given statement using the formal notation.

7. There is a triangle \( x \) such that for all squares \( y \), \( x \) is above \( y \).

8. For all circles \( x \), there is a square \( y \) such that \( y \) is to the right of \( x \).

9. For each of the following equations, determine which of the following statements are true:
   (1) For all real numbers \( x \), there exists a real number \( y \) such that the equation is true.
   (2) There exists a real number \( x \), such that for all real numbers \( y \), the equation is true.
   Note that it is possible for both statements to be true or for both to be false.
a. $x^2 - 2xy + y^2 = 0$

b. $(x - 5)(y - 1) = 0$

c. $x^2 + y^2 = -1$

In 8-9, (a) rewrite the statement formally using quantifiers and variables, and (b) write a negation for the statement.

10. Everybody loves somebody.

11. Somebody loves everybody.

12. The following is the definition for $\lim_{x \to a} f(x) = L$:

   For all real numbers $\varepsilon > 0$, there exists a real number $\delta > 0$ such that for all real numbers $x$, if $a - \delta < x < a + \delta$ and $x \neq a$ then $L - \varepsilon < f(x) < L + \varepsilon$.

   Write what it means for $\lim_{x \to a} f(x) \neq L$. In other words, write the negation of the definition.