1. A group of eight people are attending the movies together.
   a. Two of the eight insist on sitting side-by-side. In how many ways can the eight be seated together in a row?
   b. Two of the people do not like each other and do not want to sit side-by-side. Now how many ways can the eight be seated together in a row?

2.  
   a. How many integers from 0 through 99,999 contain the digit 6 exactly once?
   b. How many integers from 0 through 99,999 contain the digit 6 at least once?
   c. If an integer is chosen at random from 0 through 99,999, what is the probability that it contains two or more occurrences of the digit 6?

3. Ten points labeled A, B, C, D, E, F, G, H, I, J are arranged in a plane in such a way that no three lie on the same straight line.
   a. How many straight lines are determined by the ten points?
   b. How many of these straight lines do not pass through point A?
   c. How many triangles have three of the ten points as vertices?
   d. How many of these triangles do not have A as a vertex?

4. A student council consists of 15 students.
   a. In how many ways can a committee of six be selected from the membership of the council?
   b. Two council members have the same major and are not permitted to serve together on a committee. How many ways can a committee of six be selected from the membership of the council?
   c. Two council members always insist on serving on committees together. If they can’t serve together, they won’t serve at all. How many ways can a committee of six be selected from the council membership?

5. An instructor gives an exam with fourteen questions. Students are allowed to choose any ten to answer.
   a. How many different choices of ten questions are there?
   b. Suppose six questions require proof and eight do not.
      (i) How many groups of ten questions contain four that require proof and six that do not?
      (ii) How many groups of ten questions contain at least one that requires proof?
      (iii) How many groups of ten questions contain at most three that require proof?
   c. Suppose the exam instructions specify that at most one of questions 1 and 2 may be included among the ten. How many different choices of ten questions are there?

6. a. How many distinguishable ways can the letters of the word HULLABALOO be
arranged in order?

b. How many distinguishable orderings of the letters of HULLABALOO begin with U and end with L?

c. How many distinguishable orderings of the letters of HULLABALOO contain the two letters HU next to each other in order?

7. A lottery game offers $2 million to the grand prize winner, $20 to each of 10,000 second prize winners, and $4 to each of 50,000 third prize winners. The cost of the lottery is $2 per ticket. Suppose that 1.5 million tickets are sold. What is the expected gain or loss of a ticket?

8. The instructor of a discrete mathematics class gave two tests. Twenty-five percent of the students received an A on the first test and 15% of the students received A’s on both tests. What percent of the students who received A’s on the first test also received A’s on the second test?

9. A pool of 10 semifinalists for a job consists of 7 men and 3 women. Because all are considered equally qualified, the names of two of the semifinalists are drawn, one after the other, at random, to become finalists for the job.
   a. What is the probability that both finalists are women?
   b. What is the probability that both finalists are men?
   c. What is the probability that one finalist is a woman and the other is a man?

10. One urn contains 10 red balls and 25 green balls, and a second urn contains 22 red balls and 15 green balls. A ball is chosen as follows: First an urn is selected by tossing a loaded coin with probability 0.4 of landing heads up and probability 0.6 of landing tails up. If the coin lands heads up, the first urn is chosen; otherwise, the second urn is chosen. Then a ball is picked at random from the chosen urn.
    a. What is the probability that the chosen ball is green?
    b. If the chosen ball is green, what is the probability that it was picked from the first urn?

11. A drug-screening test is used in a large population of people of whom 4% actually use drugs. Suppose that the false positive rate is 3% and the false negative rate is 2%. Thus a person who uses drugs tests positive for them 98% of the time, and a person who does not use drugs tests negative for them 97% of the time.
   a. What is the probability that a randomly chosen person who tests positive for drugs actually uses drugs?
   b. What is the probability that a randomly chosen person who tests negative for drugs does not use drugs?